

- A function $f(x)$ is said to be periodic if there exists some $T > 0$ such that $f(x+T) = f(x)$ for all x in the domain of $f(x)$.
- In case, the T in the definition of period of $f(x)$ is the smallest positive real number then this 'T' is called the period of $f(x)$.

- **Periods of various trigonometric functions are listed below:**

- 1) $\sin x$ has period 2π
- 2) $\cos x$ has period 2π
- 3) $\tan x$ has period π
- 4) $\sin(ax+b)$, $\cos(ax+b)$, $\sec(ax+b)$, $\operatorname{cosec}(ax+b)$ all are of period $2\pi/a$
- 5) $\tan(ax+b)$ and $\cot(ax+b)$ have π/a as their period
- 6) $|\sin(ax+b)|$, $|\cos(ax+b)|$, $|\sec(ax+b)|$, $|\operatorname{cosec}(ax+b)|$ all are of period π/a
- 7) $|\tan(ax+b)|$ and $|\cot(ax+b)|$ have $\pi/2a$ as their period

- **Sum and Difference Formulae of Trigonometric Ratios**

- 1) $\sin(a + \beta) = \sin(a)\cos(\beta) + \cos(a)\sin(\beta)$
- 2) $\sin(a - \beta) = \sin(a)\cos(\beta) - \cos(a)\sin(\beta)$
- 3) $\cos(a + \beta) = \cos(a)\cos(\beta) - \sin(a)\sin(\beta)$
- 4) $\cos(a - \beta) = \cos(a)\cos(\beta) + \sin(a)\sin(\beta)$
- 5) $\tan(a + \beta) = [\tan(a) + \tan(\beta)] / [1 - \tan(a)\tan(\beta)]$
- 6) $\tan(a - \beta) = [\tan(a) - \tan(\beta)] / [1 + \tan(a)\tan(\beta)]$
- 7) $\tan(\pi/4 + \theta) = (1 + \tan \theta) / (1 - \tan \theta)$
- 8) $\tan(\pi/4 - \theta) = (1 - \tan \theta) / (1 + \tan \theta)$
- 9) $\cot(a + \beta) = [\cot(a) \cdot \cot(\beta) - 1] / [\cot(a) + \cot(\beta)]$
- 10) $\cot(a - \beta) = [\cot(a) \cdot \cot(\beta) + 1] / [\cot(\beta) - \cot(a)]$

- **Double or Triple -Angle Identities**

- 1) $\sin 2x = 2\sin x \cos x$
- 2) $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
- 3) $\tan 2x = 2 \tan x / (1 - \tan^2 x)$
- 4) $\sin 3x = 3 \sin x - 4 \sin^3 x$
- 5) $\cos 3x = 4 \cos^3 x - 3 \cos x$
- 6) $\tan 3x = (3 \tan x - \tan^3 x) / (1 - 3 \tan^2 x)$

- 1) **For angles A, B and C, we have**

- 1) $\sin(A + B + C) = \sin A \cos B \cos C + \cos A \sin B \cos C + \cos A \cos B \sin C - \sin A \sin B \sin C$
- 2) $\cos(A + B + C) = \cos A \cos B \cos C - \cos A \sin B \sin C - \sin A \cos B \sin C - \sin A \sin B \cos C$
- 3) $\tan(A + B + C) = [\tan A + \tan B + \tan C - \tan A \tan B \tan C] / [1 - \tan A \tan B - \tan B \tan C - \tan A \tan C]$
- 4) $\cot(A + B + C) = [\cot A \cot B \cot C - \cot A - \cot B - \cot C] / [\cot A \cot B + \cot B \cot C + \cot A \cot C - 1]$

- **List of some other trigonometric formulas:**

- 1) $2\sin A \cos B = \sin(A + B) + \sin(A - B)$
- 2) $2\cos A \sin B = \sin(A + B) - \sin(A - B)$
- 3) $2\cos A \cos B = \cos(A + B) + \cos(A - B)$

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- 4) $2\sin A \sin B = \cos(A - B) - \cos(A + B)$
- 5) $\sin A + \sin B = 2 \sin \left[\frac{(A+B)}{2} \right] \cos \left[\frac{(A-B)}{2} \right]$
- 6) $\sin A - \sin B = 2 \sin \left[\frac{(A-B)}{2} \right] \cos \left[\frac{(A+B)}{2} \right]$
- 7) $\cos A + \cos B = 2 \cos \left[\frac{(A+B)}{2} \right] \cos \left[\frac{(A-B)}{2} \right]$
- 8) $\cos A - \cos B = 2 \sin \left[\frac{(A+B)}{2} \right] \sin \left[\frac{(B-A)}{2} \right]$
- 9) $\tan A \pm \tan B = \frac{\sin(A \pm B)}{\cos A \cos B}$
- 10) $\cot A \pm \cot B = \frac{\sin(B \pm A)}{\sin A \sin B}$

• **Method of solving a trigonometric equation:**

- 1) If possible, reduce the equation in terms of any one variable, preferably x . Then solve the equation as you used to in case of a single variable.
- 2) Try to derive the linear/algebraic simultaneous equations from the given trigonometric equations and solve them as algebraic simultaneous equations.
- 3) At times, you might be required to make certain substitutions. It would be beneficial when the system has only two trigonometric functions.

• **Some results which are useful for solving trigonometric equations:**

- 1) $\sin \theta = \sin a$ and $\cos \theta = \cos a \Rightarrow \theta = 2n\pi + a$
- 2) $\sin \theta = 0 \Rightarrow \theta = n\pi$
- 3) $\cos \theta = 0 \Rightarrow \theta = (2n + 1)\pi/2$
- 4) $\tan \theta = 0 \Rightarrow \theta = n\pi$
- 5) $\sin \theta = \sin a \Rightarrow \theta = n\pi + (-1)^n a$ where $a \in [-\pi/2, \pi/2]$
- 6) $\cos \theta = \cos a \Rightarrow \theta = 2n\pi \pm a$, where $a \in [0, \pi]$
- 7) $\tan \theta = \tan a \Rightarrow \theta = n\pi + a$, where $a \in [-\pi/2, \pi/2]$
- 8) $\sin \theta = 1 \Rightarrow \theta = (4n + 1)\pi/2$
- 9) $\sin \theta = -1 \Rightarrow \theta = (4n - 1)\pi/2$
- 10) $\sin \theta = -1 \Rightarrow \theta = (2n + 1)\pi/2$
- 11) $|\sin \theta| = 1 \Rightarrow \theta = 2n\pi$
- 12) $\cos \theta = 1 \Rightarrow \theta = 2n\pi$
- 13) $|\cos \theta| = 1 \Rightarrow \theta = n\pi$