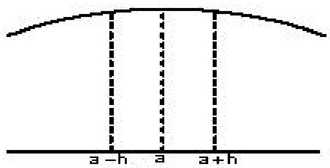
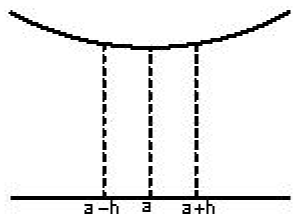



A function f(x) is said to have a local maximum at x = a if the value of f(a) is greater than all the values of f(x) in a small neighbourhood of x = a. Mathematically, f (a) > f (a – h) and f (a) > f (a + h) where h > 0, then a is called the point of local maximum.




- A function f(x) is said to have a local minimum at x = a, if the value of the function at x = a is less than the value of the function at the neighboring points of x = a. Mathematically, f (a) < f (a – h) and f (a) < f (a + h) where h > 0, then a is called the point of local minimum.
- A point of local maximum or a local minimum is also called a point of local extremum.
- A point where the graph of function is continuous and has a tangent line and where the concavity changes is called point of inflexion.
- At the point of inflexion, either y" = 0 and changes sign or y" fails to exist.
- At the point of inflexion, the curve crosses its tangent at that point.
- A function cannot have point of inflexion and extrema at the same point. Working rules to find points of local maxima and local minima:

**1. First Derivative Test:** If f'(a) = 0 and f'(x) changes its sign while passing through the point x = a, then

- f(x) would have a local maximum at x = a if f'(a – 0) > 0 and f'(a + 0) < 0. It means that f'(x) should change its sign from positive to negative.
- f(x) would have local minimum at x = a if f'(a – 0) < 0 and f'(a + 0) > 0 . It means that f'(x) should change its sign from negative to positive.
- If f(x) doesn't change its sign while passing through x = a, then f (x) would have neither a maximum nor minimum at x = a. e.g. f (x) = $x^3$ doesn't have any local maxima or minima at x = 0.

**2. Second Derivative Test:**

- Let f(x) be a differentiable function on a given interval and let f'' be continuous at stationary point. Find f'(x) and solve the equation f'(x) = 0 given let x = a, b, … be solutions.
- There can be two cases:

**Case (i)**: If f''(a) <0 then f(a) is maximum.

**Case (ii)**: If f ''(a) > 0 then f(a) is minimum.

- In case, f''(a) = 0 the second derivatives test fails and then one has to go back and apply the first derivative test.
- If f''(a) = 0 and a is neither a point of local maximum nor local minimum then a is a point of inflection.

**3. $n^{th}$ Derivative Test for Maxima and Minima:** Also termed as the generalization of the second derivative test, it states that if the n derivatives i.e. f '(a) = f''(a) = f'''(a) =……… = $f^n(a)$ = 0 and $f^{n+1}(a) \neq 0$ (all derivatives of the function up to order ‘n’ vanish and (n + 1)th order derivative does not vanish at x = a), then f (x) would have a local maximum or minimum at x = a iff n is odd natural number and that x = a would be a point of local maxima if $f^{n+1}$ (a) < 0 and would be a point of local minima if $f^{n+1}$ (a) > 0.

In some questions involving determination of maxima and minima, it might become difficult to decide whether f(x) actually changes its sign while passing through x = a and here, $n^{th}$ derivative test can be applied.

Global Minima & Maxima of f(x) in [a, b] is the least or the greatest value of the function f(x) in interval [a, b].

1. The function f(x) has a global maximum at the point ‘a’ in the interval I if f (a) ≥ f(x), for all x ∈ I.

2. Function f(x) has a global minimum at the point ‘a’ if f (a) ≤ f (x), for all x ∈ I.

- Global Maxima Minima always occur either at the critical points of f(x) within [a, b] or at the end points of the interval.
- Computation of Global Maxima and minima in maxima minima problems:

1. Compute the critical points of f(x) in (a, b). Let the various critical points be $C_1, C_2, \ldots, C_n$.

2. Next, compute the value of the function at these critical points along with the end points of the domain. Let us denote these values by $f(C_1)$, $f(C_2)$..........$f(C_n)$.

3. Now, compute $M^* = \max\{f(a), f(C_1), f(C_2)\ldots\ldots\ldots.f(C_n), f(b)\}$ and $M^{**} = \min\{f(a), f(C_1), f(C_2)\ldots\ldots\ldots.f(C_n), f(b)\}$.Now $M^*$ is the maximum value of f(x) in [a, b] and $M^{**}$ is the minimum value of f(x) in [a, b].

- In order to find global maxima or minima in open interval (a, b) proceed as told above and after the first two steps, compute

  $M_1 = \max\{f(C_1), f(C_2)\ldots\ldots\ldots.f(C_n)\}$ and

  $M_2 = \min\{f(C_1), f(C_2)\ldots\ldots\ldots.f(C_n)\}$.

- Now if x approaches $a^-$ or if x approaches $b^-$, the limit of $f(x) > M_1$ or its limit $f(x) < M_1$ would not have global maximum (or global minimum) in (a, b) but if as x approaches $a^-$ and x approaches $b^-$, $\lim f(x) < M_1$ and $\lim f(x) > M_2$, then $M_1$ and $M_2$ will respectively be the global maximum and global minimum of f(x) in (a,b).
- If *f*(*x*) is a continuous function on a closed bounded interval [*a*,*b*], then *f*(*x*) will have a global maximum and a global minimum on [*a*,*b*]. On the other hand, if the interval is not bounded or closed, then there is no guarantee that a continuous function *f*(*x*) will have global extrema.
- If *f*(*x*) is differentiable on the interval *I*, then every global extremum is a local extremum or an end point extremum.