- The derivative of f, denoted by f'(x) is given by $f'(x) = \lim_{?x \to 0}$ (? y)/(? x) = dy/dx
- The right hand derivative of f at x = a is denoted by $f'(a^+)$ and is given by $f'(a^+) = \lim_{h \to 0^+}(f(a+h)-f(a))/h$
- The left hand derivative of f at x = a is denoted by $f'(a^-)$ and is given by $f'(a^-) = \lim_{h \to 0^-} (f(a-h)-f(a))/-h$

- For a function to be differentiable at x=a, we should have $f'(a^-)=f'(a^+)$ i.e. $\lim_{h \to 0} (f(a-h)-f(a))/(-h) = \lim_{h \to 0} (f(a+h)-f(a))/h$.
- $\lim_{h \to 0} \sin 1/h$ fluctuates between -1 and 1.
- If at a particular point say x =a, we have $f'(a^+) = t_1$ (a finite number) and $f'(a^-) = t_2$ (a finite number) and if $t_1 \neq t_2$, then f' (a) does not exist, but f(x) is a continuous function at x = a.
- **Continuity and differentiability** are quite interrelated. **Differentiability** always implies continuity but the converse is not true. This means that a differentiable function is always continuous but if a function is continuous it may or may not be differentiable.

Some basic formulae:

- ❑ $\frac{d}{dx}(c) = 0$
- ❑ $\frac{d}{dx}[a\,f(x) + b\,g(x)] = af'(x) + b\,g'(x)$
- ❑ $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
- ❑ $\frac{d}{dx}[f(x)/g(x)] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
- ❑ $\frac{d}{dx}[f(x)^{g(x)}] = (f(x))^{g(x)}\left[\frac{g(x)}{f(x)} f'(x) + g'(x) ln f(x)\right]$

- If a function is not derivable at a point, it need not imply that it is discontinuous at that point. But, however, discontinuity at a point necessarily implies non-derivability.
- In case, a function is not differentiable but is continuous at a particular point say x = a, then it geometrically implies a sharp corner at x = a.
- A function f is said to be derivable over a closed interval [a, b] if :

  1.For the points a and b, $f'(a^+)$ and $f'(b^-)$ exist and

  2.For ant point c such that a < c < b, $f'(c^+)$and $f'(c^-)$ exist and are equal.

- If y = f(u) and u = g(x), then dy/dx = dy/du.du/dx = f'(g(x)) g'(x). This method is also termed as the chain rule.
- For composite functions, differentiation is carried out in this way:

If $y = [f(x)]^n$, then we put $u = f(x)$. So that $y = u^n$. Then by chain rule:

$y/dx = dy/du.du/dx = nu^{(n-1)}f'(x) = [f(x)]^{(n-1)} f'(x)$

Differential calculus problems involving parametric functions:

If x and y are functions of parameter t, first find dx/dt and dy/dt separately. Then dy/dx=(dy/dt)/(dx/dt).

- If the functions f(x) and g(x) are derivable at x = a, then the following functions are also derivable:

1.f(x) + g(x)

2.f(x) - g(x)

3.f(x) . g(x)

4.f(x) / g(x), provided g(a) ≠ 0

- If the function f(x) is differentiable at x =a while g(x) is not derivable at x = a, then the product function f(x). g(x) can still be differentiable at x = a.
- Even if both the functions f(x) and g(x) are not differentiable at x = a, the product function f(x).g(x) can still be differentiable at x = a.
- Even if both the functions f(x) and g(x) are not derivable at x = a, the sum function f(x) + g(x) can still be differentiable at x = a.
- If function f(x) is derivable at x = a, this need not imply that f'(x) is continuous at x = a.