- A function f(x) is said to be continuous at x= a if

  $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

  Thus, unlike limits, for continuity it is essential for the function to be defined at that particular point and the limiting value of the function should be equal to f(a).

- The function f(x) will be discontinuous at x =a in either of the following situations:
  a. $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist but are not equal.
  b. $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist and are equal but not equal to f(a).
  c. f(a) is not defined.
  d. At least one of the limits does not exist.
- If you are required to comment on the continuity of a function, then you may just look for the points on the domain where the function is not defined.

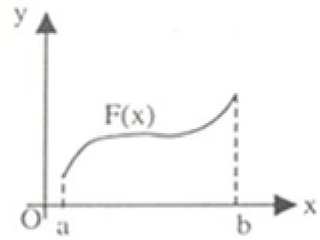
Some important properties of continuous functions:

If the functions f(x) and g(x) are both continuous at x =a then the following results hold true:

1. cf (x) is continuous at x =a where c is any constant.
2. f(x) + g(x) is continuous at x = a.
3. f(x).g(x) is continuous at x= a.
4. f(x)/g(x) is continuous at x= a, provided g(a) ≠ 0.

- If a function f is continuous in (a, b), it means it is continuous at every point of (a, b).
- If f is continuous in [a, b] then in addition to being continuous ay every point of domain, f should also be continuous at the end points i.e. f(x) is said to be continuous in the closed interval [a, b] if
  1. f(x) is continuous in (a, b)
  2. $\lim_{x\to a^+} f(x) = f(a)$

$\lim_{x\to a^-} f(x)=f(a)$



- While solving problems on continuity, one need not calculate continuity at every point, in fact the elementary knowledge of the function should be used to search the points of discontinuity.
- In questions like this where a function h is defined as

  h(x) = f(x) for a < x < b

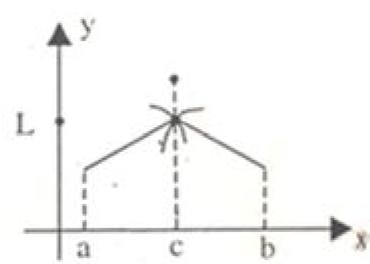
  g(x) for b < x < c

- The functions f and g are continuous in their respective intervals, then the continuity of function h should be checked only at the point x = b as this is the only possible point of discontinuity.
- If the point 'a' is finite, then the necessary and sufficient condition for the function f to be continuous at a is that $\lim_{x\to a^-} f(x)$ and $\lim_{n\to a^+} f(x)$ should exist and be equal to f(a).
- A function continuous on a closed interval [a, b] is necessarily bounded if both a and b are finite. This is not true in case of open interval.
- If the function u = f(x) is continuous at the point x=a, and the function y=g(u) is continuous at the point u = f(a), then the composite function y=(gof)(x)=g(f(x)) is continuous at the point x=a.
- Given below is the table of some common functions along with the intervals in which they are continuous:

Functions f(x)	**Interval in which f(x) is continuous**
Constant C	$(-\infty,\infty)$
b^n, n is an integer > 0	$(-\infty,\infty)$
$\lvert x-a \rvert$	$(-\infty,\infty)$
x^{-n}, n is a positive integer.	$(-\infty,\infty)-\{0\}$
$a_0x^n + a_1x^{n-1} + \dots\dots + a_{n-1}x + a_n$	$(-\infty,\infty)$

p(x)/q(x), p(x) and q(x) are polynomials in x	R - {x:q(x)=0}
sin x	R
cos x	R
tan x	R-{nπ:n=0,±1,........}
cot x	R-{(2n-1)π/2:n=0,±1,± 2,........}
sec x	R-{(2n-1)π/2:n=0,±1,± 2,........}
e^x	R
ln x	(0, ∞)

- If you know the graph of a function, it can be easily judged without even solving whether a function is continuous or not. The graph below clearly shows that the function is discontinuous.
- If $\lim_{x\to a^-} f(x) = L_1$ and $\lim_{x\to a^+} f(x) = L_2$, where L_1 and $L_2$are both finite numbers then it is called discontinuity of first kind or ordinary discontinuity.



- A function is said to have discontinuity of **second kind**if neither $\lim_{x\to a^+} f(x)$ nor $\lim_{x\to a^-} f(x)$ exist.
- If any one of $\lim_{x\to a^+} f(x)$ or $\lim_{x\to a^-} f(x)$ exists and the other does not then the function f is said to have mixed discontinuity.
- If $\lim_{x\to a} f(x)$ exists but is not equal to f(a), then f(x) has removable discontinuity at x = a and it can be removed by redefining f(x) at x=a.
- If $\lim_{x\to a} f(x)$ does not exist, then we can remove this discontinuity so that it becomes a non-removable or essential discontinuity.
- A function f(x) is said to have a jump discontinuity at a point x=a if, $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ and f(x) and may be equal to either of previous limits.
- The concepts of limit and continuity are closely related. Whether a function is continuous or not can be determined by the limit of the function.