- $z_1 = a+ib$ and $z_2 = c+id$ then $z_1 = z_2$ implies that $a = c$ and $b = d$.
- If we have a complex number z where $z = a+ib$, the conjugate of the complex number is denoted by z^* and is equal to $a-ib$. In fact, for any complex number z, its conjugate is given by $z^* = Re(z) – Im(z)$.
- Division of complex numbers: The numerator as well as denominator should first be multiplied by the conjugate of the denominator and then simplified.

Argument of a complex number:

- Argument of a complex number p(z) is defined by the angle which OP makes with the positive direction of x-axis.
- Argument of z generally refers to the principal argument of z (i.e. the argument lying in $(–\pi, \pi)$ unless the context requires otherwise.
- Hence, the argument of the complex number $z = a + ib = r (\cos \theta + i \sin \theta)$ is the value of θ satisfying $r\cos \theta = a$ and $r \sin \theta = b$.
- The angle θ is given by $\theta = \tan^{-1} |b/a|$.

The value of argument in various quadrants is given below:

$$\varphi = \arg(z) = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0 \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0 \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0 \\ \frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0 \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0 \\ \text{indeterminate} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

- If $OP = |z|$ and $\arg (z) = \theta$, then obviously $z = r (\cos \theta + i \sin \theta)$ and is called the polar form of complex number z.
- $|(z-z_1) / (z-z_2)| =1$ the locus of point representing z is the perpendicular bisector of line joining z_1 and z_2.
- $-|z| \leq Re(z) \leq |z|$ and $-|z| \leq Im(z) \leq |z|$
- If a and b are real numbers and z_1 and z_2 are complex numbers then

$$|az_1 + bz_2|^2 + |bz_1 - az_2|^2 = (a^2 + b^2) (|z_1|^2 + (|z_2|^2)$$

- The distance between the complex numbers z_1 and z_2 is given by $|z_1 - z_2|$.
- In parametric form, the equation of line joining z_1 and z_2 is given by $z = tz_1 + (1-t)z_2$.
- If $A(z_1)$ and $B(z_2)$ are two points in the argand plane, then the complex slope μ of the straight line AB is given by $\mu = (z_1 - z_2)/ ({}_1 - {}_2)$.
- **Two lines having complex slopes μ_1 and μ_2 are:**

1. Parallel iff $\mu_1 = \mu_2$
2. Perpendicular iff $\mu_1 = -\mu_2$ or $\mu_1 + \mu_2 = 0$

- If $A(z_1)$, $B(z_2)$, $C(z_3)$ and $D(z_4)$ are four points in the argand plane, then the angle θ between the lines AB and CD is given by $\theta = \arg\{(z_1 - z_2)/(z_3 - z_4)\}$

Some basic properties of complex numbers:

(i) $||z_1| - |z_2|| = |z_1+z_2|$ and $|z_1-z_2| = |z_1| + |z_2|$ iff origin, z_1, and z_2 are collinear and origin lies between z_1 and z_2.

(ii) $|z_1 + z_2| = |z_1| + |z_2|$ and $||z_1| - |z_2|| = |z_1-z_2|$ iff origin, z_1 and z_2 are collinear and z_1 and z_2 lie on the same side of origin.

(iii) The product of n^{th} roots of any complex number z is $z(-1)^{n-1}$.

(iv) $amp(z^n) = n \, amp \, z$

(v) The least value of $|z - a| + |z - b|$ is $|a - b|$.

(vi) $\overline{(z_1 \pm z_2)} = \bar{z}_1 + \bar{z}_2$

(vii) $\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$

(viii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2} \; (\bar{z}_2 \neq 0)$

Demoivre's Theorem: The theorem can be stated in two forms:

Case I: If n is any integer, then

(i) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(ii) $(\cos\theta_1 + i\sin\theta_1) \cdot (\cos\theta_2 + i\sin\theta_2) \ldots\ldots\ldots (\cos\theta_n + i\sin\theta_n) = \cos(\theta_1 + \theta_2 + \theta_3 \ldots\ldots\ldots + \theta_n) + i\sin(\theta_1 + \theta_2 + \ldots\ldots\ldots\ldots\; \theta_n)$

Case II: For p and q such that $q \neq 0$, we have

$(\cos\theta + i\sin\theta)^{p/q} = \cos((2k\pi + pq)/q) + i\sin((2k\pi+pq/q)$ where $k = 0,1,2,3,\ldots\ldots,q-1$

- Demoivre's formula does not hold for non-integer powers.
- Main application of Demoivre's formula is in finding the n^{th} roots of unity. So, if we write the complex number z in the polar form then,

$z = r(\cos x + i\sin x)$

then $z^{1/n} = [r(\cos x + i\sin x)]^{1/n}$

$= r^{1/n} [\cos(x+2k\pi/n) + i \sin(x+2k\pi/n)]$

- Here k is an integer. To get the *n* different roots of *z* one only needs to consider values of *k* from 0 to $n - 1$.
- Continued product of the roots of a complex quantity should be determined using theory of equations.
- The modulus of a complex number is given by $|z| = \sqrt{x^2+y^2}$.
- The only complex number with modulus zero is the number (0, 0).

The following figures illustrate geometrically the meaning of addition and subtraction of complex numbers: