- **If x, y ∈ R and n ∈ N, then the** binomial theorem **states that**

  $x+y)^n = {}^nC_0x^n + {}^nC_1 x^{n-1}y + {}^nC_2 x^{n-2} y^2 + \ldots\ldots \ldots .. + {}^nC_rx^{n-r} y^r + \ldots.. + {}^nC_ny$

  which can be written as $\boldsymbol{\Sigma {}^nC_rx^{n-r}y^r}$. **This is also called as the binomial theorem formula which is used for solving many problems.**

- **Some chief properties of binomial expansion of the term $(x+y)^n$:**
  1. The number of terms in the expansion is (n+1) i.e. it is one more than the index.
  2. The sum of indices of x and y is always n.
  3. The binomial coefficients of the terms which are equidistant from the starting and the end are always equal.The simple reason behind this is
  4. C(n, r) = C(n, n-r) which gives C(n, n) C(n, 1) = C(n, n-1) C(n, 2) = C(n, n-2).
- **Such an expansion always follows a simple rule which is:**
  1. The subscript of C i.e. the lower suffix of C is always equal to the index of y.
  2. Index of x = n – (lower suffix of C).
- The $(r+1)^{th}$term in the expansion of expression $(x+y)^n$ is called the general term and is given by $T_{r+1} = \boldsymbol{{}^nC_rx^{n-r}y^r}$
- The term independent of x is obviously without x and is that value of r for which the exponent of x is zero.
- The middle term of the binomial coefficient depends on thevalue of n. There can be two different cases according to whether n is even or n is odd.
  1. If n is even, then the total number of terms are odd and in that case there is a single middle term which is $(n/2 +1)^{th}$ and is given by ${}^nC_{n/2}\, a^{n/2}\, x^{n/2}$.
  2. On the other hand, if n is odd, the total number of terms is even and then there are two middle terms $[(n+1)/2]^{th}$ and $[(n+3)/2]^{th}$ which are equal to ${}^nC_{(n-1)/2}\, a^{(n+1)/2}\, x^{(n-1)/2}$ and ${}^nC_{(n+1)/2}\, a^{(n-1)/2}\, x^{(n+1)/2}$
- The binomial coefficient of the middle term is the greatest binomial coefficient of the expansion.
- Some of the standard binomial theorem formulas which should be memorized are listed below:
  1. $\boldsymbol{C_0 + C_1 + C_2 + \ldots.. + C_n} = 2^n$
  2. $\boldsymbol{C_0 + C_2 + C_4 + \ldots.. = C_1 + C_3 + C_5 + \ldots\ldots\ldots} = 2^{n-1}$
  3. $\boldsymbol{C_0^2 + C_1^2 + C_2^2 + \ldots.. + C_n^2} = {}^{2n}\boldsymbol{C_n = (2n!)/ n!n!}$
  4. $\boldsymbol{C_0C_r + C_1C_{r+1} + C_2C_{r+2} + \ldots.. + C_{n-r}C_n = (2n!)/ (n+r)!(n-r)!}$

5. Another result that is applied in binomial theorem problems is $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
6. We can also replace $^mC_0$ by $^{m+1}C_0$ because numerical value of both is same i.e. 1. Similarly we can replace ^mC_m by $^{m+1}C_{m+1}$.
7. Note that $(2n!) = 2^n.\ n!\ [1.3.5. \ldots (2n-1)]$

- In order to compute numerically greatest term in a binomial expansion of **$(1+x)^n$,** find $T_{r+1} / T_r = (n - r + 1)x/r$. Then put the absolute value of x and find the value of r which is consistent with the inequality $T_{r+1} / T_r > 1$.
- If the index n is other than a positive integer such as a negative integer or fraction, then the number of terms in the expansion of $(1+x)^n$ is infinite.
- The expansions in ascending powers of x are valid only if x is small. If x is large, i.e. $|x| > 1$ then it is convenient to expand in powers of $1/x$ which is then small.
- The binomial expansion for the nth degree polynomial is given by:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

- **Following expansions should be remembered for $|x| < 1$:**

1. $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \ldots\ldots \infty$
2. $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots\ldots \infty$
3. $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots\ldots \infty$
4. $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \ldots\ldots \infty$